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MODIFIED DYNAMIC MATRIX CONTROL

Su Whan Sung and In-Beum Lee†

Department of Chemical Engineering, School of Environmental Engineering,
Pohang University of Science and Technology, San 31 Hyoja Dong, Pohang 790-784, Korea
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Abstract – DMC (Dynamic Matrix Control) has been used successfully in industry for the last decade. It can deal with constraints and unusual dynamic behavior directly. It also shows a good control performance for the servo problem. Relatively, it can't reject disturbances systematically. We propose a modified DMC method to control the regulatory process more efficiently. The proposed DMC method makes the control output by subtracting the estimated disturbance from the control output of the original DMC. Here, the disturbance is estimated by a new disturbance estimator. It shows better control performances than the original DMC.

Key words: *DMC, Estimator, FIR, Regulation*

INTRODUCTION

During the last decade, DMC has been used successfully in process control. DMC proposed by Cutler and Ramaker [1980] can treat an unusual dynamic behavior of the process efficiently. An analytical solution to the DMC control strategy can be obtained because it is based on unconstrained control. QDMC (Quadratic Dynamic Matrix Control) [Garcia and Morshedi, 1986] explicitly considers process constraints on the process output and the manipulated variable. Garcia and Morari [1982]'s theorems imply that a small control horizon contributes to the robustness of the process. Maurath et al. [1988] discussed dynamic properties (for example, stability and robustness according to the controller parameter choices) of SISO and closed-loop systems with predictive controllers. Georgiou et al. [1988] mentioned the control performance of DMC for high-purity distillation columns and suggested a simple nonlinear version of DMC which uses a coordinate transformation. Freedman and Bhatia [1985] proposed an adaptive DMC with on-line evaluation of the model coefficients. Maiti et al. [1994] used DMC to neutralize a continuous process stream in a stirred tank and discussed the deteriorated control performance because of the process/model mismatch. To overcome this difficulty, they proposed an adaptive DMC strategy using a new on-line closed-loop identification scheme.

DMC uses an additive disturbance concept to compensate for the effects of disturbance. That is, it is assumed that the present prediction error is same as the future disturbance. However, if the disturbance passes through the dynamics of the process, this assumption can be true only when a steady state is obtained. Therefore, even though DMC shows a good control performance for the set point change process, an equivalent performance can't be obtained for the input disturbance rejection process. Therefore, we propose a modified DMC to control the regulatory process more efficiently. Here, the control output is determined by subtracting the estimated disturbance from the control output of the original DMC, where, the disturbance is estimated by a new disturbance estimator.

DISTURBANCE ESTIMATOR AND PROPOSED CONTROL STRATEGY

Assume that a disturbance and the process have the following pattern.

$$a_n \frac{d^n y(t)}{dt^n} + a_{n-1} \frac{d^{n-1} y(t)}{dt^{n-1}} + \cdots + a_1 \frac{dy(t)}{dt} + a_0 y(t) = b_m \frac{d^m [u(t)+d(t)]}{dt^m} + b_{m-1} \frac{d^{m-1}[u(t)+d(t)]}{dt^{m-1}} + \cdots + b_1 \frac{d[u(t)+d(t)]}{dt} + u(t) + d(t) \tag{1}$$

and then the output of the model is as follows.

$$a_n \frac{d^n y_m(t)}{dt^n} + a_{n-1} \frac{d^{n-1} y_m(t)}{dt^{n-1}} + \cdots + a_1 \frac{dy_m(t)}{dt} + a_0 y_m(t) = b_m \frac{d^m u(t)}{dt^m} + b_{m-1} \frac{d^{m-1} u(t)}{dt^{m-1}} + \cdots + b_1 \frac{du(t)}{dt} + u(t) \tag{2}$$

From (1) and (2), we can obtain the following equation.

$$a_n \frac{d^n p_e(t)}{dt^n} + a_{n-1} \frac{d^{n-1} p_e(t)}{dt^{n-1}} + \cdots + a_1 \frac{dp_e(t)}{dt} + a_0 p_e(t) = b_m \frac{d^m d(t)}{dt^m} + b_{m-1} \frac{d^{m-1} d(t)}{dt^{m-1}} + \cdots + b_1 \frac{dd(t)}{dt} + d(t) \tag{3}$$

where $p_e(t)=y(t)-y_m(t)$ denotes the prediction error.

Therefore, if disturbance is constant during the sampling time and the process is open-loop stable, the following equation can be obtained directly.

$$P_{e,k} = \sum_{i=1}^{T} h(k)\, d(k-i) \tag{4}$$

where h(k) and T denote the impulse response and model

†To whom correspondence should be addressed.

horizon, respectively and k represents the present sampling data. Then, d(k − 1) can be estimated from the following recursive equation.

$$d(k-1)=\frac{P_{e,k}-\sum_{i=2}^{T}h(k)\,d(k-i)}{h(1)} \tag{5}$$

If the process has time delay, the above equation should be modified as follows.

$$d(k-D-1)=\frac{P_{e,k}-\sum_{i=D+2}^{T}h(k)\,d(k-i)}{\sum_{i=1}^{D+1}h(k)} \tag{6}$$

where D denotes the number of the sampling time corresponding to the assumed time delay. It should be noted that the denominator of (6) is not h(D+1) but a summation form. The reason is as follows. If D_p(the number of the sampling time corresponding to the real process time delay) isn't equal to the process time delay over sampling time or the sampling time is very small, $h(D_p+1)$ can be a very small value and then $h(D_p+1)$ can't be used as the denominator of (6) because the noise effects can be amplified. In this case, we assume that the time delay [D of (6)] is D_p+1. That is, h(1)=⋯=h(D+1)=0 and h(D+2) should be replaced by $h(D_p+2)+h(D_p+1)$. Similarly, if we assume that the time delay [D of (6)] is D_p+2, then h(1)=h(2)=⋯=h(D+2)=0 and h(D+3) should be substituted with $h(D_p+3)+h(D_p+2)+h(D_p+1)$. Therefore, the denominator term of (6) should be the summation form. We recognized from the results of simulation that as the modeling error is large, D should be amplified. From (6), we can estimate disturbance d(k − D − 1) easily. Then, we get the control output by subtracting d(k − D − 1) from the control output of the original DMC as follows.

$$u(k)_{MDMC}=u(k)_{DMC}-d(k-D-1)w$$
$$0\le w\le 1 \tag{7}$$

where u and D denote the control output and the number of the sampling time corresponding to the assumed time delay. Subscripts MDMC and DMC denote the modified DMC and the original DMC, respectively and w is the tuning parameter. Here, the prediction error is estimated with the following equation.

$$p_e(k)=y(k)-\sum_{i=1}^{T}h(k)u_{MDMC}(k-i) \tag{8}$$

where y(k) denotes the process output.

When w=0, the MDMC is the same as the DMC controller. So, we can infer that if a small w value is chosen, the modification of (7) does not degrade the robustness of the DMC. Also, from extensive simulation studies, as the modeling error is severe, w value should be small to improve the robustness to plant/model mismatches.

If D is T-1 and the disturbance prediction transfer function of Wellons and Edgar [1987] is the process model, the proposed disturbance estimator is the same as that of Wellons and Edgar [1987] because it is assumed that the disturbance is a step input. Because Wellons and Edgar [1987]'s method is based on ARX(Auto-Regressive model with an eXogenous input) model, their good performance in disturbance prediction can't be guaranteed for the impluse model of DMC.

Lundström et al. [1995] used a white noise filtered through first order dynamics (a stochastic disturbance model) as the disturbance model regardless to the process dynamics to improve the disturbance rejection performance. Also, they proposed a prior setting value to incorporate the type of the disturbance. On the other hand, we use the process model as the disturbance model. May be, it is more practical that the deterministic disturbance is assumed to be filtered through the process dynamics.

For MIMO (Multi-input Multi-output) systems, this idea can also be applied easily. For example, we can construct the following disturbance estimator for the 2×2 system.

$$MD=MH^{-1}ME \tag{9}$$

$$MD=\begin{bmatrix}d_1(k-D_1-1)\\ d_2(k-D_2-1)\end{bmatrix} \tag{10}$$

$$MH=\begin{bmatrix}\sum_{i=1}^{D_1+1}h_{11}(k) & \sum_{i=1}^{D_2+1}h_{12}(k)\\ \sum_{i=1}^{D_1+1}h_{21}(k) & \sum_{i=1}^{D_2+1}h_{22}(k)\end{bmatrix} \tag{11}$$

$$ME=\begin{bmatrix}p_{1,e}(k)-\sum_{i=D_1+2}^{T}h_{11}(k)d_1(k-i)-\sum_{i=D_2+2}^{T}h_{12}(k)d_2(k-i)\\ p_{2,e}(k)-\sum_{i=D_1+2}^{T}h_{21}(k)d_1(k-i)-\sum_{i=D_2+2}^{T}h_{22}(k)d_2(k-i)\end{bmatrix} \tag{12}$$

where D_1 and D_2 denote the assumed time delay term of disturbances [$d_1(k)$ and $d_2(k)$] to $y_1(k)$ and $y_2(k)$, respectively.

Prediction error can be obtained from the following equation.

$$p_{e,1}(k)=y_1(k)-\sum_{i=1}^{T}h_{11}(k)u_{MDMC,1}(k-i)-\sum_{i=1}^{T}h_{12}(k)u_{MDMC,2}(k-i)$$

$$p_{e,2}(k)=y_2(k)-\sum_{i=1}^{T}h_{21}(k)u_{MDMC,1}(k-i)-\sum_{i=1}^{T}h_{22}(k)u_{MDMC,2}(k-i) \tag{13}$$

The control strategy is as follows.

$$\begin{bmatrix}u_{MDMC,1}(k)\\ u_{MDMC,2}(k)\end{bmatrix}=\begin{bmatrix}u_{DMC,1}(k)\\ u_{DMC,2}(k)\end{bmatrix}-w\begin{bmatrix}d_1(k-D_1-1)\\ d_2(k-D_2-1)\end{bmatrix} \tag{14}$$

It should be noted that constraints on the manipulated variable can be considered directly by substituting u(k+i) − d(k − D − 1) for u(k+i) of the original DMC. Therefore, the proposed strategy can be easily implemented in various DMC-type controllers to improve the input disturbance rejection.

SIMULATION STUDIES AND DISCUSSIONS

Consider the following distillation column model originally described by Wood and Berry [1973].

$$\begin{bmatrix}y_1(s)\\ y_2(s)\end{bmatrix}=\begin{bmatrix}\frac{12.8\exp(-s)}{16.7s+1} & \frac{-18.9\exp(-3s)}{21.0s+1}\\ \frac{6.6\exp(-7s)}{10.9s+1} & \frac{-19.4\exp(-3s)}{14.4s+1}\end{bmatrix}$$

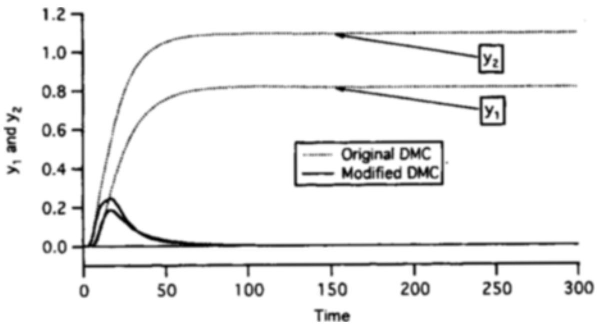

Fig. 1. Control results of the modified DMC and DMC for disturbance rejection when a ramp disturbance is introduced.

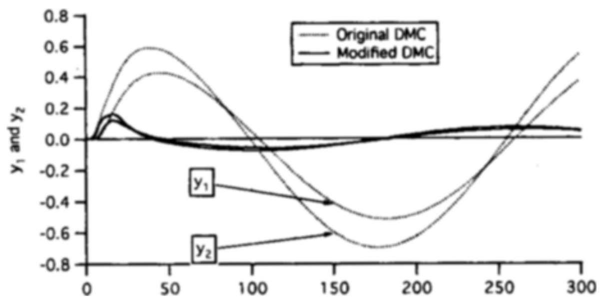

Fig. 2. Control results of the modified DMC and DMC for disturbance rejection when a sinusoidal disturbance is introduced.

$$\begin{bmatrix} u_1(s) \\ u_2(s) \end{bmatrix} + \begin{bmatrix} \dfrac{3.8\exp(-8s)}{14.9s+1} \\ \dfrac{4.9\exp(-3s)}{13.2s+1} \end{bmatrix} d(s) \tag{15}$$

where sampling time=1, control horizon=1, prediction horizon=10, w=1.0, T=102, D_1=2, D_2=4. And the object function used in the original DMC is as follows.

$$J(\Delta u) = E^T E \tag{16}$$

where E denotes the predicted error term.

Two kinds of disturbances are considered as follows.

$$d(n)=\sin(0.02n) \tag{17}$$

$$d(n)=0.03n \tag{18}$$

Simulation results are shown in Fig. 1 and 2. From the simulation results, we can recognize that the proposed method produces superior control performances to the original DMC for regulatory processes. Fig. 1 shows that the original DMC can't reject the ramp disturbance (18) but relatively, the proposed control strategy gives a good control result. In Fig. 2, low frequency disturbance (17) can not be rejected efficiently by the original DMC. Contrarily, the proposed method can incorporate it more efficiently.

Assume that the process is changed to the following process. Here, the process (19) and the model (15) have parameter mismatches up to 33.33 %.

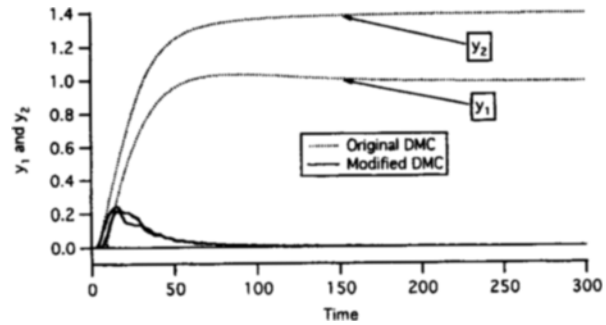

Fig. 3. Control results of the modified DMC and DMC with modeling errors for disturbance rejection when a ramp disturbance is introduced.

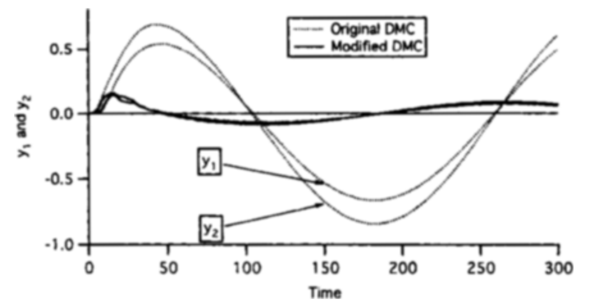

Fig. 4. Control results of the modified DMC and DMC with modeling errors for disturbance rejection when a sinusoidal disturbance is introduced.

$$\begin{bmatrix} y_1(s) \\ y_2(s) \end{bmatrix} = \begin{bmatrix} \dfrac{10.8\exp(-s)}{13.7s+1} & \dfrac{-15.9\exp(-4s)}{25.0s+1} \\ \dfrac{5.6\exp(-6s)}{13.9s+1} & \dfrac{-15.4\exp(-3s)}{10.4s+1} \end{bmatrix}$$

$$\begin{bmatrix} u_1(s) \\ u_2(s) \end{bmatrix} + \begin{bmatrix} \dfrac{3.8\exp(-8s)}{14.9s+1} \\ \dfrac{4.9\exp(-3s)}{13.2s+1} \end{bmatrix} d(s) \tag{19}$$

Simulation results are shown in Fig. 3 and 4 for the ramp and sinusoidal disturbances, respectively. From the results of the above simulations and many other simulation studies, we can recognize that the proposed method shows better disturbance rejection performances compared with the original DMC in the presence of modeling errors. Also, the proposed DMC shows acceptable robustness to measurement noises and modeling errors.

CONCLUSIONS

We proposed a modified DMC to reject disturbances more efficiently. The proposed method uses a new disturbance predictor and can be implemented easily in various DMC-type predictive controllers. The proposed control strategy shows a superior control performance to DMC for several simulated processes and an acceptable robustness to modeling errors.

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NOMENCLATURE

d : disturbance
D : the number of sampling time corresponding to the assumed time delay
E : predicted error
h : impulse response
p_e : prediction error
T : model horizon
t : time
u : controller output
w : tuning parameter of the modified DMC
y : process output

Subscripts

DMC : dynamic matrix control
e : error
m : model
MDMC : modified dynamic matrix control
p : process

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